Vortex and dipole solitons in defect lattices

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Abstract:
In this talk, band gap structures and nonlinear stability properties of localized optical solitons that arise in the solution of the nonlinear Schrödinger (NLS) equation with irregular lattice-type potentials will be presented.
The governing equation for the physical model is defined as the nonlinear Schrödinger (NLS) equation with an external potential in Eq. (1)

$$iu_z + \Delta u + |u|^2 u - V(x,y)u = 0. \tag{1}$$

In optics, $u(x,y,z)$ corresponds to the complex-valued, slowly varying amplitude of the electric field in the xy plane propagating in the z direction, $\Delta u \equiv u_{xx} + u_{yy}$ corresponds to diffraction, the cubic term in u originates from the nonlinear (Kerr) change of the refractive index and $V(x,y)$ is an external optical potential that can be written as the intensity of a sum of N phase-modulated plane waves,

$$V(x,y) = \frac{V_0}{N^2}\left|\sum_{n=0}^{N-1} e^{i\vec{k}_n.\vec{x}+i\theta_n(x,y)}\right|^2 \tag{2}$$

where $V_0 > 0$ is the peak depth of the potential, i.e., $V_0 = max_{x,y}V(x,y)$, $\vec{x} = (x,y)$, $\vec{k}_n$ is a wave vector defined by $(k_x^n, k_y^n) = [K\cos(2\pi n/N), K\sin(2\pi n/N)]$, and $\theta(x,y)$ is a phase function through which irregularities are introduced.
As external potential, we consider a lattice with an edge dislocation and a lattice with a vacancy defect. These lattices obtained from Eq. (2), and given by Eq. (3) and Eq. (4) respectively.

$$V(x,y) = \frac{V_0}{25}[2cos(k_x x + \theta(x,y)) + 2cos(k_y y) + 1]^2 \tag{3}$$

with the phase-dislocation function $\theta(x,y) = \frac{3\pi}{2} - tan^{-1}(\frac{y}{x})$.

$$V(x,y) = \frac{V_0}{25}\left|2cos(k_x x) + 2cos(k_y y) + e^{i\theta(x,y)}\right|^2 \tag{4}$$

with the phase function $\theta(x,y) = tan^{-1}(\frac{y-y_0}{x}) - tan^{-1}(\frac{y+y_0}{x})$.
In this study, solution of the NLS equation with an external potential is obtained by using the spectral renormalization method.
In the final part of the presentation, effect of the defects on the first nonlinear band gap structures, and the nonlinear stability properties of the dipole and vortex solitons in defect lattices will be discussed.

References:

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